C2 • Exploring functions involving fractional and negative powers of x

Mathematical goals

To develop learners' ability to:

- find the stationary points of a function and determine their nature;
- solve appropriate equations in order to find the intercepts of a function.

To encourage learners to:

- connect the mathematical properties of a function and relate them to the graph.

Starting points

Learners should understand that $f'(x) = 0$ finds stationary points and $f''(x)$ determines their nature. Learners should also be familiar with fractional and negative indices.

Materials required

For each learner you will need:

- mini-whiteboard.

For each small group of learners you will need:

- at least one large sheet of paper for making a poster;
- felt tip pens;
- graphic calculator (optional).

Time needed

At least 45 minutes.

Suggested approach

Beginning the session

Use mini-whiteboards to do a quick revision of differentiating functions with fractional and negative indices such as:

$$y = \sqrt{x} \qquad y = \frac{1}{x} \qquad y = \frac{3}{x^2} \qquad y = \frac{1}{2\sqrt{x}}$$

Working in groups (1)

Write the following list of functions on the board.

Set A	Set B	Set C
$f(x) = x^2 + \frac{8}{x}$	$f(x) = x + \frac{2}{x} + 3$	$f(x) = \sqrt{x} - 3 + \frac{2}{\sqrt{x}}$
$f(x) = x + \frac{1}{x^2}$	$f(x) = \sqrt{x} - x$	$f(x) = 1 - \frac{1}{x} - \frac{2}{x^2}$
$f(x) = x + \frac{8}{x^2}$	$f(x) = 2\sqrt{x} - x$	$f(x) = \sqrt{x} - x^2$
$f(x) = x^2 - \frac{8}{x}$		
$f(x) = x^2 + \frac{1}{x}$		

Ask learners to work in pairs and ask each pair to choose a function. Explain that set A are normal challenge, set B are medium challenge and set C are high challenge. The extra challenge tends to come from the equations that need to be solved and manipulated.

Ask them to explore their chosen function (i.e. find out all they can about the intercepts, stationary points, and their nature) and then draw a good quality sketch of the function based on this information.

Explain that all their rough work, working out and thinking must be written on the large sheet of paper, which will become a poster.

Remind learners that crossings out and mistakes are all part of the learning.

As the pairs are working on their functions, go round and ask them to talk you through what they are doing. Note on the board any problems or interesting points that arise, for use in whole group discussion at the end of the session.

If learners finish early, they can check their graph on a graphic calculator or computer and consider what happens to the shape of the graph as x approaches infinity and justify why this is so from the equation.

Reviewing and extending learning

When all pairs have found sufficient information to sketch their graph, ask them to show their poster and share the findings. Ask each pair to talk about something that they found either difficult or interesting. Also, discuss any points that you have noted on the board.

What learners might do next

Learners could look at integrating these functions. They could then link integrals with areas using limits. Relating these to their graphs will help them to realise that, because of the breaks in the graph, not all limits are possible.

Further ideas

This idea can be applied to any type of function, e.g. quadratics and cubics.

It can also be used to investigate the effects of transformations on a function rather than stationary points and intercepts, e.g. trigonometrical or modulus functions.

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